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WORKING GROUP ON SOCIAL SCIENCE METHODS IN DEFENSE ANALYSIS

A RULE-BASED DIAGNOSTIC SYSTEM DEPENDENT UPON UNCERTAINTY MEASURES

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ABSTRACT

This paper considers the modeling of a general diagnostic system based upon mathematical-logical considerations. The heart of the system consists of input data, predetermined error distributions or matching tables, and inference rules formulated within a general fuzzy set system framework. Applications to the multiple target data association and other military problems are outlined.

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1. INTRODUCTION

Problems often arise which are not easily treated from either a deterministic or probabilistic viewpoint. This situation typically occurs when knowledge of all joint probability distributions of the set of modeling parameters of interest is unobtainable, and thus only a relatively low level of information is present. One example of this is the problem of modeling the most appropriate error or matching distributions with respect to a fixed collection of of ship classifications obtained from experts in the field. These classifications may well be overlapping and vague in concept. Such typically linguistic information gleaned from these individuals tends to indicate simple models for the distributions which do not take into account compound or joint occurences of classifications. This is because, as good as human beings are as integrators of disparate information, there is a limit to the quantity and level of information that can be handled over a given time. Indeed, in such problems as classification, the number of joint event occurences to be considered, in general, increases exponentially, unless unlikely combinations can be efficiently ruled out.

As a consequence of the above discussion, there appears a need to establish a systematic approach to the quantification and use of such low level information. The paper presented here consists of three basic aspects:

First, a logical basis is presented for utilizing a mixture of possibilistic and probabilistic modeling techniques for dealing with military and other problems involving natural language descriptions or other incomplete numerical or statistical quantities. This is based upon earlier work where such descriptions were shown essentially to correspond to classes of random subsets of domains of attributes. (See [1-4].)

Second, a comprehensive diagnostic procedure is developed which utilizes generalized error distributions and inference rules connecting groups of attributes or symptoms with possible values of an unknown parameter, or equivalently, possible diagnoses of possible faults. Some applications of this to military situations, including the multiple target data association problem, are given. (See [5-8] for previous work in this area.)

Third, the problem of modeling the interface between natural language inputs and the main diagnostic procedure is briefly treated.

2. LOGICAL BASIS FOR UTILIZATION OF A MIXTURE OF PROBABILISTIC AND FUZZY SET UNCERTAINTY MEASURES

The procedure presented in this paper is based upon three general theoretical mathematical-logical results obtained previously by the author in somewhat different forms:

(a) Fuzzy sets and their operators correspond in a natural way to random sets and their operators such that fuzzy set (or possibilistic) modeling in effect is a weakened form of probabilistic modeling, thus allowing for interchange between the two types of approaches. This result leads to the procedure where all input information to a problem may be converted separately to fuzzy set forms connected by ordinary two-valued logic truth functions-often, conjunction. In turn, the well developed fuzzy set calculus [9] may be used to simplify the computations leading to a final possibility distribution - or equivalently, a single fuzzy set description (through its membership function) of the unknown parameter of interest. An open problem of great interest involves the many-to-one relation between random set models equivalent to a given fuzzy set model (in a sense to be made more precise in the ensuing technical discussion): which particular random set representation to choose for a given fuzzy set model and how much information is lost when a particular random set description is replaced by a fuzzy set one?

(b) Given input information consisting of an ordinary logical combination of fuzzy set ones for an unknown parameter of interest (the parameter may well be multidimensional in form), a uniformly most accurate pure fuzzy set description exists which is obtainable by replacement of all ordinary two-valued truth connectors by corresponding fuzzy set ones. This description can be shown under sufficient conditions of smoothness of behavior to yield an asymptotically consistent estimator of the parameter in question with computable error bounds. This result forms the basis for the structure of the diagnostic system as applied to the multiple target data association or "correlation" problem: the PACT (Possibilistic Approach to Correlation and Tracking) algorithm. (See [7] and [8].)

(c) Under very general conditions, conditional fuzzy sets may be constructed, analagous to conditional random variables and vectors. In turn, this leads to a fuzzy set form of Bayes' Theorem. (See also [9] and [10].) Then, with the identification of inference rules with posterior distributions of the parameter of interest and error distributions-or matching tables- with posterior data distributions, the uniformly most accurate estimator, mentioned in (b), is essentially the same as the overall posterior estimator of the parameter in the fuzzy set Bayesian sense. (See [7] and [8].)

Some detailed technical descriptions of the above three types of results justifying the establishment of the diagnostic system will now be given.

A. LOGICAL BASIS FOR (a)

FUZZY SET SYSTEMS IN GENERAL

Although it is not possible to condense fuzzy set theory in terms of all of its major thrusts here, some relevant highlights can be touched upon. The basic building block is the membership function

$$\phi_A : X \to [0,1] \quad , \qquad (1)$$

defining fuzzy subset A of base space X.

By making the range of Φ_A be a subset of $\{0,1\}$, A becomes a set in the ordinary sense. Operations among fuzzy susets of a base space extend those of ordinary subsets of the space. For example, one can define fuzzy intersection between two fuzzy sets by use of the pointwise operator min applied to the corresponding membership functions. On the other hand, one could just as well define other operations on fuzzy sets which might also reasonably be called fuzzy intersection since they also reduce to ordinary intersection when the fuzzy sets involved are also ordinary ones. One such example is the operator prod (for pointwise product operating on the corresponding membership functions). Similarly for fuzzy union, max or probsum (probability sum, where probsum$(a,b) \triangleq a+b-ab = 1-(1-a)(1-b)$) can serve as definitions, from among an infinity of choices. Consider also fuzzy complement. A natural choice is the operator $1-(\cdot)$, but as in the above cases, many other different definitions could be used. Which ones to choose? Obviously, this basic problem must impinge upon all uses of fuzzy set theory; a partial solution to this will be briefly considered below. (See also Goodman [4].)

However the problem of obtaining fuzzy set membership functions is relatively simple, provided that the domain of discourse or base space is properly specified. For example, the fuzzy set representing the attribute "tall" clearly must be some nondecreasing or monotone increasing function over its domain. But the slope and increase is dependent upon whether "tall" refers to adult males now living in the Unites States, or to immature females who resided in India during the eighteenth century, or to ships, etc. Using proper sampling or survey techniques in conjunction with suitable parameterization, analogous to that employed in modeling probability distributions, empirical membership functions may also be constructed. (See the survey of procedures by Dubois and Prade [9] 1980, pp. 255-264.)(Another modeling approach to fuzzy set membership functions can be through the empirical one point coverage functions of the equivalent random sets, the latter topic to be discussed later.)

One approach to the problem outlined above concerning nonuniqueness[1] of fuzzy set definitions is as follows: First, attempt to abstract the essential meaning-and no more than that- of complement (or negation), intersection (or conjunction), and union (or disjunction). In the case of the last two operators, a natural family of operators has been proposed and investigated by some researchers: the t-norms and t-conorms, respectively. (See Klement [12] and Goodman [4], for further details.) Then for any triple of operators of interest

$$F \triangleq (\psi_n, \psi_\&, \psi_{or}), \qquad (2)$$

where ψ_n is usually chosen to be $1-(\cdot)$ for complementation (though not necessarily so restricted), $\psi_\&$ is a t-norm, and ψ_{or} is a t-conorm, compound fuzzy set definitions may also be defined, with structure not dependent on the specific choice of F. This leads to unified definitions for implication, equivalence, universal and existential quantifiers, subset relations, projections, and general functions and arithmetic operations, among many other concepts. Multivalued logic, as a formal extension of ordinary two-valued logic plays the central role in the above constructions. (See Goodman [4] for an example of this approach to the construction of general fuzzy set systems.) Second, determine from theoretical considerations which subcollection of fuzzy set systems F leads to interpretation in terms of probability theory. As mentioned later, two families (the semi-distributive DeMorgan and the larger class, the J-copula DeMorgan) can be chosen for possible F. Specifically, these are characterized by their weak homomorphic relations to corresponding random set systems. "Weak" as used above means that equality as is usually used in the concept of homomorphism is replaced by (the weaker) equality with respect to one point coverage probabilities. Finally, use empirical procedures such as moment matching techniques to determine the most appropriate F from the reduced collection. (See also section 4 for further comments.)

CONNECTIONS BETWEEN FUZZY SET SYSTEMS AND RANDOM SETS

The next set of results comprise type (a) basis for the correlation algorithm, where fuzzy set and random set descriptions may be interchanged (not without some information loss or increase) See Goodman, [4], [6], for background and mathematical details.

Define for any fuzzy subset A of X,

$$S_U(A) = \Phi_A^{-1}[U,1] \qquad (3)$$

$S_U(A)$ is a random subset of X with all outcomes being nested with respect to each other, where U is any random variable uniformly distributed over $[0,1]$. Note the special case when Φ_A is monotone and the relation to r.v.'s. Extend the above definition, by considering any stochastic process $\underset{\sim}{U} \triangleq (U_j)_{j\in J}$ of uni-

1. Menes [11] has proposed a unifying theory of uncertainty modeling which contains as special cases fuzzy set theory with $\psi_\&$=min and ψ_{or}=max, probability theory, and topological neighborhood theory.

form r.v.'s over $[0,1]$ which is also a J-copula, i.e., all joint marginal distributions depend in form only on the number of distinct arguments. In turn, it follows that for any collection $\underset{\sim}{A} \stackrel{d}{=} (A_j)_{j\in J}$ of fuzzy subsets A_j of base space X_j, $j\in J$,

$$S_{\underset{\sim}{U}}(\underset{\sim}{A}) \stackrel{d}{=} (S_{U_j}(A_j))_{j\in J} \qquad (4)$$

is a well defined random subset (of appropriate X_j's) process.

Theorem 1.
Let $\underset{\sim}{U}$ be arbitrary as above and define the fuzzy set operator $\psi_{\&}$ by, for any $\underset{\sim}{u} \stackrel{d}{=} (u_j)_{j\in J}$, $u_j\in[0,1]$, $j\in J$,

$$\psi_{\&}(\underset{\sim}{u}) \stackrel{d}{=} \Pr(\underset{j\in J}{\&}(U_j \le u_j)), \quad (5)$$

noting that $\psi_{\&}$ will be well defined and the same as when defined recursively. Let ψ_{or} be the DeMorgan transform of $\psi_{\&}$, i.e.,

$$\psi_{or}(u,v) = 1-\psi_{\&}(1-u,1-v), \qquad (6)$$

for all $u,v\in[0,1]$. Then let F denote any corresponding fuzzy set system formed from these definitions for $\psi_{\&}$ and ψ_{or}. Then:

System F is weak homomorphic, separately for all three operators, to the natural corresponding random set system through $S_{\underset{\sim}{U}}$.

Thus, for example, for fuzzy set intersection defined through $\psi_{\&}$,

$$\textcircled{\cap}\underset{\sim}{A} \approx S_U(\textcircled{\cap}\underset{\sim}{A}) \approx \cap S_U(\underset{\sim}{A}), \qquad (7)$$

where $\textcircled{\cap}$ denotes fuzzy intersection and $\underset{\sim}{A}$ is an arbitrary collection of fuzzy subsets of X. The equivalence relation $\approx$ is defined by the one point coverage probabilities, in the case of random sets, and membership values, in the case of fuzzy sets. Thus eq.(7) is the same as

$$\phi_{\textcircled{\cap}\underset{\sim}{A}}(x) = \Pr(x\in S_U(\textcircled{\cap}\underset{\sim}{A})) = \Pr(x\in \cap S_U(\underset{\sim}{A})), \qquad (7')$$

for all $x\in X$ ∎

Remarks

(i) S_U has the property that for any base space X and any fuzzy subset A of X,

$$A \approx S_U(A). \qquad (8)$$

Such $\underset{\sim}{S}$ mapping is called a canonical choice function. S_U is called a choice function family induced by S_U. Note that there can be infinitely many such families induced by the same canonical choice function, as is the case here, if different joint distributions can be constructed for the random sets involved.

(ii) Another canonical choice function T can be constructed by identifying $T(A)$ with its ordinary set membership function-which is also random-where all $\phi_{T(A)}(x)$'s are statistically independent zero-one random variables with $\Pr(\phi_{T(A)}(x) = 1) = \phi_A(x)$, all $x\in X$.

In turn, choose first any semi-distributive DeMorgan fuzzy set system F - a semi-distributive system satisfies a form of distributivity formally similar to the intersection expansion of the probability of a union of events; any such DeMorgan system, letting $\psi_n = 1-(\cdot)$, has for its last two components, (min,max), (prod,probsum), or more generally any ordinal sum-a certain type of linear like combination-of these two. (See Goodman [4] and Klement [12].) Then define the choice function family $\underset{\sim}{T}$ by using the technique as above for constructing T, but expanded in terms of another index involving $\psi_{\&}$ from F. This family yields weak homomorphic relations for $\psi_{\&}$.

(iii) For the special cases for $\underset{\sim}{U}$, as above, if $U_j = U$, for all $j\in J$, or all U_j's are statistically independent, and similarly, if $\psi_{\&}$ = prod in the construction of $\underset{\sim}{T}$, then both $S_{\underset{\sim}{U}}$ and $\underset{\sim}{T}$ yield not only for the corresponding system F to have weak homomorphic random counterparts, but also a wide variety of other homomorphic-like relations.

(iv) Other choice function families may be constructed yielding for semidistributive systems weak homomorphic relations for arbitrary combinations of $\psi_{\&}$ and ψ_{or}, as well as for fuzzy arithmetic operations.

Theorem 2.
Given any ordinary n-ary operator over a collection of power classes of base spaces and any choice function family , there exists a unique n-ary fuzzy set operator which is weak homomorphic to the ordinary one over the random sets induced through the choice function family. The latter operator is an extension of the former. All results can be explicitly constructed. ∎

Thus, the conclusions from Theorems 1 and 2 emphasize that fuzzy sets may be identified with classes of random sets equivalent under the one point coverage functions to the former. These random sets may differ consider-

ably, according to the choice function employed generating them, such as the nested S_U type and the very broken-up T type. Many fuzzy set operators correspond weak homomorphically to natural corresponding ordinary random set operators, which are also not uniquely determined as is the case for random sets relative to equivalent fuzzy sets, although by specifying both random set operator and choice function family, the weak homomorphic fuzzy set is uniquely determined and is an extension of the former.

Finally, some recent results of some importance will be mentioned [10] :

(i) There is only one possible nested random set (one point coverage, i.e. $\approx$) equivalent to any given fuzzy set A, namely, $S_U(A)$.

(ii) Fuzzy sets which admit equivalent random intervals have been characterized.

(iii) For any finite space, the maximal entropy random subset equivalent to a given fuzzy subset A is T(A). $S_U(A)$ may or may not be the minimal entropy equivalent random subset with respect to A, depending on further restrictions on the form of A.

B. LOGICAL BASIS FOR (b)

Theorem 3. Uniformly Most Accurate Estimators

Suppose that information concerning unknown parameter Q consists of the following forms:

(i) Data $\hat{Z}$.

(ii) Matching tables M_k, k=1,..,m.

(iii) Relations R_t, t=1,..,r.

Let g: $[0,1]\times\cdots\times[0,1] \longrightarrow [0,1]$ be nondecreasing (m+r factors)

with respect to the partial ordering of vectors. In particular, g can be any t-norm, the natural operator corresponding to conjunction ("and"). (see [4]).

Define the possibility distribution Φ by

$$\Phi(Q\,|\,\hat{z}) \stackrel{d}{=} \underset{(\text{all } Z)}{1\text{-}g(1\text{-}C(Z,\hat{Z},Q))}\ , \qquad (9)$$

where it is assumed g is extendable to an arbitrary number of arguments (this is guaranteed if, e.g., g is symmetric and associative, which will be the case if g is a t-norm) , and

$$C(Z,\hat{Z},Q) \stackrel{d}{=} g(\ R(Z,Q),\ M(\hat{Z},Z))\ , \qquad (10)$$

$$M(\hat{Z},Z) \stackrel{d}{=} \underset{(k=1,..,m)}{g(M_k(\hat{Z}_k,Z_k))}$$

= matching table effect under g , (11)

$$R(Z,Q) \stackrel{d}{=} \underset{(t=1,..,r)}{g(R_t(Z,Q))}$$

= relation effect under g , (12)

and Q is arbitrary $\in$ dom(Q).

For any confidence levels

$$\underset{\sim}{\alpha} \stackrel{d}{=} (\alpha_1,\alpha_2,\ldots,\alpha_m)\ , \qquad (13)$$

$$\underset{\sim}{\beta} \stackrel{d}{=} (\beta_1,\beta_2,\ldots,\beta_r)\ , \qquad (14)$$

with α_k, $\beta_t \in [0,1]$, all k,t , define the original hypothesis set as

$$H_o(\underset{\sim}{\alpha},\underset{\sim}{\beta};g) \stackrel{d}{=} \{\tbinom{Z}{Q} \mid \mathop{\&}_{t=1}^{r}(R_t(Z,Q)\ge\beta_t)\ \&\ \mathop{\&}_{k=1}^{m}(M_k(\hat{Z}_k,Z_k)\ge\alpha_k)\}.\ (15)$$

Then (for $\hat{Z}$ fixed), for any possibility distribution $D(\binom{Z}{Q}|\hat{Z})$ as a function of Q over dom(Q) , Z over dom(Z), yields the smallest set

$$\{\tbinom{Z}{Q} \mid D(\tbinom{Z}{Q}|\hat{Z}) \ge g(\underset{\sim}{\alpha},\underset{\sim}{\beta})\} \supseteq H_o(\underset{\sim}{\alpha},\underset{\sim}{\beta};g),\ (16)$$

simultaneously for all possible $\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$, when D is chosen

$$D(\tbinom{Z}{Q}|\hat{Z}) = C(Z,\hat{Z},Q)\ , \qquad (17)$$

for all $Z,\hat{Z},Q$. In turn, Φ enjoys a similar property with respect to the projection 1-g(1--) applied to H_o and D.

(For proofs, see [10] , section 10.) ■

Thus, the above theorem exhibits in a general setting the uniformly most accurate single fuzzy set description of Q, given $\hat{Z}$ and g.

For details concerning the asymptotic consistency of Φ given in eq.(9), see [10], section 11.

C. LOGICAL BASIS FOR (c)

The next theorem displays the concept of a conditic al fuzzy set. In turn, Theorem 5 is a fuzzy set analogue of the classical probabilistic Bayes' theor Theorem 6 shows that the optimal estimator (i.e., t uniformly most accurate fuzzy set description as given in Theorem 3) may be also considered to be a fuzzy set Bayesian one.

Theorem 4 (Related to Goodman [3],[4].)
For any fuzzy subset C of $X_1 \times X_2$ and system F, the X_1 projection $C_{(1)}$ of C into X_1 is

$$\phi_{C_{(1)}}(x_1) \overset{d}{=} \underset{x_2 \in X_2}{\psi_{or}}(\phi_C(x_1,x_2)), \quad (18)$$

for any $x_1 \in X_1$. Similarly, for $C_{(2)}$ w.r.t. X_2.

For any $x_j \in X_j$, j=1,2, there exist fuzzy subset $C_{(1|x_2)}$ of X_1 and fuzzy subset $C_{(2|x_1)}$ of X_2 such that

$$\phi_C(x_1,x_2) = \psi_\&(\phi_{C_{(1|x_2)}}(x_1), \phi_{C_{(2)}}(x_2))$$
$$= \psi_\&(\phi_{C_{(2|x_1)}}(x_2), \phi_{C_{(1)}}(x_1)). \quad (19)$$

If $\psi_\&$ is monotone increasing in all of its arguments, then the conditional fuzzy sets $C_{(1|x_2)}$ and $C_{(2|x_1)}$ are uniquely determined. ∎

Theorem 5 Fuzzy Bayes' Theorem (Goodman [3])
Suppose that a fuzzy subset B of X_1 is given, calling B the prior set, and for each $x_1 \in X_1$, there is a fuzzy subset C_{x_1} of X_2 indexed by $x_1 \in X_1$, called the conditional data (on parameter) set. Then there is a fuzzy subset D of $X_1 \times X_2$, such that $D_{(1)} = B$, $D_{(2|x_1)} = C_{x_1}$; for all $x_1 \in X_1$, and such that $D_{(1|x_2)}$ (and D) are determined implicitly through eq.(19) in terms of B and $C_{x_1}; x_1 \in X_1$.

$D_{(1|x_2)}$ is called the posterior set (conditioned on x_2). ∎

This result has been used to develop a theory of fuzzy set sampling. See Goodman [3] for properties of fuzzy posterior sets for both small and asymptotically large samples.

Theorem 6. Posterior Form for Optimal Estimator

Suppose the same conditions holds as in Theorem 3. Then ϕ as given in eq.(9) is the posterior possibilistic distribution of Q given $\hat{Z}$ (see [7]) where the following identifications are made:

$$M(\hat{Z},Z) = poss(Z \mid \hat{Z}), \quad (20)$$

$$R(Z,Q) = poss(Q \mid Z), \quad (21)$$

and the sufficiency condition

$$poss((Q \mid \hat{Z}) \mid (Z \mid \hat{Z})) = poss(Q \mid Z), \quad (22)$$

holds for all $Z, \hat{Z}, Q$, and "poss" refers to any possibility function (conditional form) constructed in accordance with its corresponding variables, using possibilistic Bayes Theorem ([7]).

(Proof: Simply use the relations

$$poss(\binom{Q}{Z} \mid \hat{Z}) = poss(\binom{(Q\mid\hat{Z})}{(Z\mid\hat{Z})})$$

$$= g(\,poss((Q \mid \hat{Z}) \mid (Z \mid \hat{Z}))\,,\ poss(Z \mid \hat{Z})) \quad (23)$$

and then apply the projection operator to both sides with respect to variable Z.) ∎

The above results lead to the following procedure:

Procedure

Given a collection of confidence statements $\underset{\sim}{C}$ about an unknown parameter Q with some statements C_j representing random sets and others representing fuzzy sets, convert the random set statements to their corresponding fuzzy set forms resulting from their one point coverage probabilities and then apply Theorem 3 or any of its extensions discussed above. Alternatively, $\underset{\sim}{C}$, by appropriate choice functions can be converted to pure random set forms. In either situation obviously a change in information content occurs. (An open research issue involves the measurement of this change.)

3. GENERAL DIAGNOSTIC SYSTEMS

In this section we direct the procedure mentioned at the end of the last section, motivated by logical bases (a), (b), (c), to general diagnostic systems. Essentially, this amounts to the breaking up of confidence statements C_j concerning parameter Q into two groups: matching tables M_k and inference rules R_t as given in Theorem 3. In summary, the relevant information mentioned above can be conveniently divided up into three parts :

1. Observed data.
2. Prior known distributions of matches between observed and true attribute values.
3. Prior known relations between the levels of matching outcomes for any attribute or group of attributes

Then, some function or statistic (in the extended sense to include possibilities as well as probabilities) of the relevant information -or of part of the information, such as only involving the first two categories listed above- is sought which will estimate the unknown parameter Q.

Let attributes A_1 , $A_2,\ldots,A_m$ be m types of information over which observed data $\hat{Z}$ can be categorized. Thus we write in partitioned form

$$\hat{Z} = \begin{pmatrix}\hat{Z}_1\\ \vdots \\ \hat{Z}_m\end{pmatrix} \qquad (24)$$

where $\hat{Z}_k$ is observed from the domain of A_k , $dom(A_k)$, for k=1,...,m. It is assumed $dom(A_k)$ is known. Corresponding to $\hat{Z}_k$ we denote as a variable Z_k any possible value $\hat{Z}_k$ could have taken in $dom(A_k)$; similarly for Z.

Let Q denote the unknown parameter vector of interest. Denote the matching table (or by a simple transform, the error distribution) for attribute A_k by M_k. Typically, M_k is evaluated as a number between 0 and 1 :

$$0 \le M_k(\hat{Z}_k, Z_k) \le 1 \qquad (25)$$

Define symbolically R_t to correspond to the t^{th} fuzzy relation connecting any Z with Q. Specifically,

$$R_t : \bigtimes_{v=1}^{h_t} dom(A_{k_v}) \times dom(Q) \longrightarrow [0,1], \qquad (26)$$

where $1 \le k_1 < k_2 < \cdots < k_{h_t} \le m$ represents the collection of attributes involved in the t^{th} relation R_t . Typically, R_t is evaluated (clearly, as a membership function) as a number between 0 and 1 :

$$0 \le R_t(Z,Q) \le 1 \quad , \qquad (27)$$

with some abuse of subscript notation. Note that formally M_k and R_t are possibility distributions (or equivalently, fuzzy set membership functions).

We may think of M_k corresponding to the following linguistic description :

$M_k(\hat{Z}_k, Z_k)$ = possibility that Z_k is the true value, when data $\hat{Z}_k$ is observed, noting both Z_k and $\hat{Z}_k \in dom(A_k)$. (28)

Similarly, we may interpret

$R_t(Z,Q)$ = possibility that Z (through attributes $A_{k_1},\ldots,A_{k_{h_t}}$) and Q are related. (29)

Often relations R_t are in the form of <u>inference rules</u> concerning the intensity or degree to which if a group of attributes match between potential observed and true values, then a restriction holds on particular possibility distribution (i.e., fuzzy set membership function may be assumed, for the unknown parameter. The example below concerning the application to the correlation problem will clarify this. Finally, utilize the computations in Theorem 3 to obtain the possibility distribution (posterior) of Q as given in eq.(9).

<u>Correlation Problem</u>

As an example of the above statements, consider the following four attributes which are commonly involved in informational inputs relative to tracking: A_1= class, A_2= frequency of signal at its source, A_3 = ship mode, and A_4 = geolocation with confidence ellipse. The natural domains of values of these attributes are typically: $dom(A_1) = \{C_1,\ldots,C_m\}$, each C_k a label for a category of ship ; $dom(A_2)$= interval $[0,M_0]$, where M_0 is some suitably chosen upper bound (in hz.); $dom(A_3) = \{D_1,\ldots,D_n\}$, each D_k being a label for a mode of operation, noting the highly overlapping flavor in general possessed by the C_k's and D_k's, where some could actually represent subcategories with respect to others; $dom(A_4) = \{(p,E_p) \mid p$ any point in R^2, E_p any confidence ellipse centered at p ; each E_p has the same fixed probability level$\}$. Next, let i and j represent two fixed track histories. That is, each letter represents a collection of data from possibly several different sensor and intelligence sources which is assumed to correspond to the same (usually unknown) target source. This data may be classified into the four types of attributes mentioned above. In addition, it is assumed that error distributions - or equivalently, matching level tables- are obtainable for each of the types of observed data. Finally, it is assumed that prior known relations are available connecting the intensities of matches between any possible outcomes of attribute categorized data between i and j and consequential levels of correlation between i and j. Usually, the latter is in the form of inference rules. Both matching tables and inference rules may be obtained either analytically , using physics and geometrical constraints, or empirically, through the establishment of a panel of experts. The term "distribution" as used above may refer to classical probabilistic or possibilistic/fuzzy set definitions. (See [9] for a survey and summary of possibilistic distributions and properties.) Then, some statistic (in the general sense) is sought which will estimate the unknown correlation level between i and j , based upon the available data, matching tables, and inference rules.

Consider first a set of confusable track histories $\{1,2,\ldots,q\}$,say . Pick out any $i \ne j$, and define, omitting the obvious subscript dependency,

$$Q \overset{d}{=} \text{poss(i and j correlate, i.e., belong to the same target source)}, \qquad (30)$$

Let all of the fuzzy relations here be of the form of inference rules. Thus, linguistically, a typical R_t corresponds to the phrase

"If a match between i and j occurs relative to attribute A_{k_1} to intensity level α_{k_1} and,..., and a match between i and j occurs relative to attribute $A_{k_{n_t}}$ to intensity level $\alpha_{k_{n_t}}$,

then i and j correlate to intensity $\xi(\underline{\alpha}_t)$",

where $\xi(\underline{\alpha}_t)$ is a number between 0 and 1 and $\underline{\alpha}_t$ is the vector of α_k's; in general, both of these values are obtained from a panel of experts.

The intensities of the attribute matches is most easily translated by an exponentiation process applied to the appropriate attribute matching functions. A simple conversion table between the degree of matching, expressed linguistically or initially numerically on a scale from 0 (no match) to 0.5 (normal match) up to 1.0 (complete match), might be established by use of the relation

$$((x)) \overset{d}{=} x/(1-x) \quad , \qquad (31)$$

for all $x \in [0,1]$, where $((x))$ is to be used as an exponent. Other translations of the intensities of matches are of course possible and may be more appropriate, following empirical studies. (Future work will consider this problem. See also Dubois and Prade [9], pp. 256-264 for similar problems.)

Combining all of the above remarks, a reasonable possibilistic model for inference rule t is

$$R_t(Z,Q) = \Psi_{\phi}(G_t(Z),\ Q^{((\xi(\underline{\alpha}_t)))}), \qquad (32a)$$

$$G_t(Z) \overset{d}{=} \underset{(v=1,..,n_t)}{g}\left(M_{k_v}(Z_{k_v}^{(i)}, Z_{k_v}^{(j)})^{((\alpha_{k_v}))}\right), \qquad (32b)$$

$$\Psi_{\phi}(x,y) \overset{d}{=} 1-g(x,1-y)\ ;\ g = \Psi_{\&} . \qquad (32c)$$

In this case, data vector $\overset{\circ}{Z}$ (and similarly for variable Z) is broken up into the i-data and j-data, as indicated by the appropriate superscript, with the previous notation still holding for the attribute indices.

Based on three general results ((a),(b),(c)), the PACT algorithm has been developed which treats the multiple target correlation problem, including data categorized as nongeolocational. Figure 1. succinctly summarizes the structure of the algorithm, which depends functionally on the collection of relevant inference rules chosen as well as the attribute matching tables.

A summary of the PACT algorithm is given below in Fig. 1 :

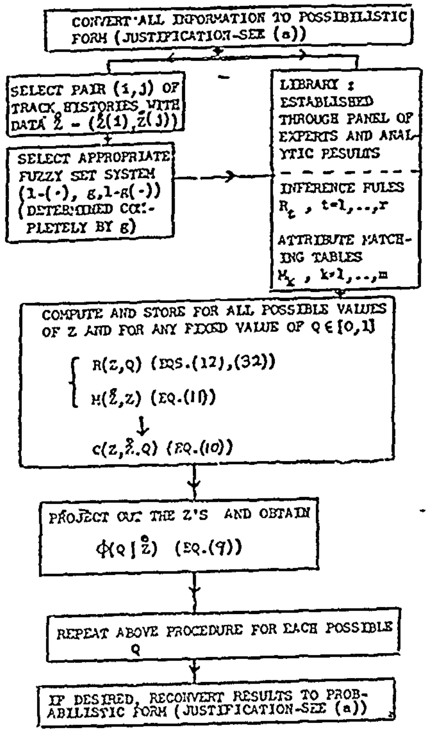

Fig. 1. Outline of the basic correlation algorithm.

A number of problems have arisen in the implementation of the PACT algorithm:

(1) How should attributes be chosen? What systematic procedures are available for determining from available experts and other informational sources what are the most important and distinct attributes to consider. Nowakowska's clustering-like approach [13] or alternatively a modified factor anaysis approach might lead to satisfactory choices.

(ii) In utilizing a panel of experts, the way questions are formulated is critical. Consequently, use of questionnaire and psychometric techniques to extract maximal unbiased information is necessary.

(iii) Perhaps the most critical problem is the actual determination of the inference rules. Even with a relatively few attributes used or a basis, there are myriad combinations of possible intensities of attribute matches leading to the corresponding inference rules. Thus, a method is needed to generate inference rules which are relatively distinct (too many redundant-like rules will cause unnecessary computer running time without adding much information content). Can a metric be designed which determines the amount of "distinctness" between rules? The answer to these problems may well lie within the purview of Artificial Intelligence techniques or related search theory procedures.

(iv) Complete flow charts for the PACT algorithm in its general form have been made (and are available to interested readers upon request). Preliminary numerical runs indicate a long running program. Consequently, by utilizing the basic bounding property of t-norms and t-conorms (see, e.g., [10], section 4), an algorithm may be obtained which is simpler in form than the original PACT algorithm and which yields as outputs lower bounds to the posterior correlation distribution.

4. MODELING NATURAL LANGUAGE DESCRIPTIONS IN FUZZY SET NOTATION

As mentioned previously, one of the apparent assets of the diagnostic system established in the previous section is the ability to handle and integrate natural language descriptions of an unknown parameter with numerical or statistical descriptions, due to the conversion of all input information into fuzzy set form. The last statement involves the assumption that all relevant linguistic information can be converted in some reasonable way to fuzzy set form. Some examples in which arguments can be established for the fuzzy set representation of sentences are:

1. x is a large number.
2. y is much larger than x.
3. "Mike is much taller than most of his close friends" seems to be true.
4. The probability that this urn contains many more black balls than white is not very high.
5. The possibility that the ship's classification is of type C when type A is observed is 0.4.
6. The probability that position x is correct given position y is observed is 0.6.
7. If two track histories (suitably updated to a common present time) are such that their geolocations match closely, in a weighted statistical sense and their classifications only moderately overlap, then the possibility that they correlate is rather low.

See [9], [14], [15] for a number of other examples. ([9] also contains extensive references to the area of fuzzy logic and approximate reasoning.). In all of the above examples, relatively primative attributes may be obtained which can be built upon, by use of appropriate fuzzy set operators, to yield back a model of the original sentence restructured in complete fuzzy set form. Thus, sentence 1, probably the simplest, is replaced by the structure $\phi_{\text{Large}}(x)$ or $\phi_L(x) \geq \alpha$, for some variable α representing the confidence level in the truth value of the sentence. (See Theorem 3, eqs. (13)-(17) for justification of this approach, instead of the more common fuzzy set approach outlined in [9] or [14].) Sentence 2 can be described in a fuzzy set context as $\phi_M(\phi_{L\text{-than}}(x,y)) \geq \alpha$, where $\phi_M : [0,1] \to [0,1]$ is some appropriate mapping representing intensification of an attribute by "very" or "much more". Some candidates for this are $\phi_M(z) = z^a$, or $\phi_M(z) = z+a$, for some appropriately chosen constant a, $a > 0$. Analagous to the modeling of statistical variables, ϕ_M, as well as $\phi_{L\text{-than}}$ could be parameterized with, where required, the relevant parameters evaluated through en estimation/empirical procedure. (Again, see [9], especially section 4.1 for such techniques.) Sentence 3 involves use of fuzzy cardinalities, since "most", a counting concept, is involved. A reasonable model for it is given as follows:

$$\phi_{\text{seem}}\left\{\phi_{\text{most}}\left(\sum_{\substack{x\in \\ \text{set of} \\ \text{all living} \\ \text{humans}}}\left\{\psi\left(\phi_{\text{much}}\left(\phi_{\substack{\text{taller} \\ \text{than}}}(\text{Mike},x)\right), \phi_{\substack{\text{very} \\ \text{(or close)}}}\left(\phi_{\substack{\text{friend} \\ \text{of}}}(x,\text{Mike})\right)\right)\right\} \Big/ \sum_{\substack{x\in \\ \text{set of} \\ \text{all living} \\ \text{humans}}}\left\{\phi_{\substack{\text{very} \\ \text{(or close)}}}\left(\phi_{\substack{\text{friend} \\ \text{of}}}(x,\text{Mike})\right)\right\}\right)\right\} \geq \alpha \qquad (33)$$

Sentence 4 is a mixture of probability and possibilities. Sentence 5 is an example of a matching table evaluation as discussed in the previous sections of this paper. Similarly, sentence 7 is an example of an inference rule evaluation. Sentence 6 is an example of a linguistic description of a pure probabilistic statement. Symbolization of these last 4 sentences can be completely carried out, but for reasons of brevity will not be displayed here. Note, in regard to sentence 6, all numerical or probabilistic sentences may be put in linguistic form without losing meaning, but, in general, require long symbolic forms. In turn, the fuzzy set descriptions of these "non-purelinguistic" forms coincides with the original mathematical symbolism. In a similar manner, symbolization may be extended to reflect tense, mood, verbal relations and various semantical connectors, by careful consideration of the primative attributes involved and pertinant variables, such as time, type of measurement, degree of intensification involved, etc. That language, linguistic descriptions, syntax, and semantics are very difficult areas to model from a comprehensive rigorous viewpoint, is attested to by the many different competing approaches found in the literature since Chomsky's ground breaking work. (See, e.g. [16], [17], [18].) What is

needed here is a general existance theorem (formulated from a rigid viewpoint) that measures the degree of faithfulness a particular fuzzy set format has with respect to the original linguistic descriptions. For further comments and results in this area see the companion paper [19]. (See also the comments in Section 2A of this paper.)

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